

COGNITIVELY GUIDED INSTRUCTION

Levels of Development of Problem Solving

- Level 1:** Direct Modeling only
- Level 2:** Both modeling and counting strategies, reflecting the structure of the problem
- Level 3:** Counting strategies, more flexible strategies such as counting up for subtraction
- Level 4:** Number facts with occasional counting strategies

Classification of Word Problems

- Join:** a direct or implied action in which a quantity is increased by a particular amount over time

Result Unknown

Robin had 5 toy cars. Her parents gave her 2 more toy cars for her birthday. How many toy cars did she have then?

Change Unknown

Robin had 5 toy cars. Her parents gave her some more toy cars for her birthday. Then she had 7 toy cars. How many toy cars did Robin's parents give her for her birthday?

Start Unknown

Robin had some toy cars. Her parents gave her 2 more toy cars for her birthday. Then she had 7 toy cars. How many toy cars did Robin have before her birthday?

- Separate:** a direct or implied action in which a quantity is decreased by a particular amount over time

Result Unknown

Colleen had 8 guppies. She gave 3 guppies to Roger. How many guppies does Colleen have left?

Change Unknown

Colleen had 8 guppies. She gave some guppies to Roger. Then she had 5 guppies left. How many guppies did Colleen give Roger?

Start Unknown

Colleen had some guppies. She gave 3 guppies to Roger. Then she had 5 guppies left. How many guppies did Colleen have to start with?

Part-Part-Whole: a static relationship among a particular set and its two disjoint subsets with no direct or implied action and no change over time

Whole Unknown

Six boys and 4 girls were playing soccer. How many children were playing soccer?

Part Unknown

Ten children were playing soccer. Six were boys and the rest were girls. How many girls were playing soccer?

Compare: comparison of two distinct, disjoint sets with no direct or implied action and no change over time

Difference Unknown

Mark has 3 mice (Reference set).
Joy has 7 mice (Compared set).
Joy has how many more mice than Mark? (Difference)

Compared Set Unknown

Mark has 3 mice (Reference set).
Joy has 4 more mice than Mark (Difference).
How many mice does Joy have? (Compared set)

Referent Unknown

Joy has 7 mice (Compared set).
She has 4 more mice than Mark (Difference).
How many mice does Mark have? (Reference set)

Direct Modeling Strategies

Strategy

Description

Addition (for $3 + 5 = _ _ _$)

Counting All Using objects or fingers, a set of 3 objects and set of 5 objects are constructed. The sets are joined and the union of the two sets is counted.

Subtraction (For $8 - 3 = _ _ _$ or $3 + _ _ _ = 8$)

Separating From Using objects or fingers, a set of 8 objects is constructed. 3 objects are removed. The answer is the number of remaining objects.

Separating To A set of 8 elements is counted out. Elements are removed from it until the number of elements remaining is equal to 3. The answer is the number of elements removed.

Adding On A set of 3 elements is constructed. Elements are added to this set until there is a total of 8 elements. The answer is found by counting the number of elements added.

Matching A set of 3 objects and a set of 8 objects are matched one to one until one set is used up. The answer is the number of objects remaining in the unmatched set.

Other (for $_ _ _ + 3 = 8$ or $_ _ _ - 3 = 8$)

Trial and Error A set of set objects is constructed. A set of 3 objects are added to or removed, and the resulting set is counted. If the final count is 8, then the number of elements in the initial set is the answer. If it is not 8, a different initial set is tried.

Counting Strategies

Strategy

Description

Addition (for $3 + 5 = \underline{\quad} \underline{\quad}$)

The counting sequence begins with 3 and continues on 5 counts. The answer is the last term in the counting sequence.

The counting sequence begins with 5 and continues on 3 counts. The answer is the last terms in the counting sequence.

Subtraction (For $8 - 3 = \underline{\quad} \underline{\quad}$ or $3 + \underline{\quad} \underline{\quad} = 8$)

A backward counting sequence is initiated starting with 8. The sequence contains 3 counting number words. The last number in the counting sequence is the answer.

A backward counting sequence starts with 8 and continues until 8 is reached. The answer is the number of words in the counting sequence.

A forward counting sequence starts with 3 and continues until 5 is reached. The answer is the number of counting words in the sequence.

Either Separating From or Counting Up From Given is used depending upon which is more efficient.

Order of Difficulty of Problems

**Level at
Which the
Problem Can
Be Readily
Solved**

Problem Type

1a	Join (Result Unknown) Separate (Result Unknown) Part-Part-Whole (Whole Unknown)
1b	Join (Change Unknown)
1b/2	Compare (Difference Unknown) Part-Part-Whole (Part Unknown)
3	Separate (Start Unknown) Join (Start Unknown)