## SOLUTIONS

### Scenario 1: Heather and Tina – Solution

Tina solved the three additive tasks presented to her and the evidence suggests that she counted from one when doing so. Each of the three additive tasks involved two screened collections and it is clear that Tina did not count perceptual items (that is visible items) on her solutions. This suggests that Tina was at Stage 2: Figurative Counting. In the case of the missing subtrahend tasks, Tina did not display a general strategy for solving these tasks. Solving the introductory task of '3 to 2' was not a sufficient indicator of a more advanced strategy. Similarly, in solving the task of '6 to 4' her solution seemed to depend on the four counters being unscreened. There is no indication that she used the counting-down-to strategy. In similar vein, Tina did not use counting-down-from to solve the removed items tasks. She was able to solve the introductory task and her solution to the last task seemed to depend on the three counters being unscreened. Thus Tina is judged to be at Stage 2: Figurative Counting.

little indication that Jack can use a range of non-count-by-ones strategies. Thus Jack is judged to be at Stage 4: Intermediate Number Sequence.

## Scenario 3: Wendy and Bill – Solution

Bill used a range of non-count-by-ones strategies to solve additive and subtractive tasks. He used the known fact of 5 + 5 to solve 5 and 4 and similarly 10 + 5 to solve 8 and 5. In solving 9 and 6 he seemed to be able to regard nine as three threes and six as two threes. He used the known addition fact of 6 + 4 = 10 to work out the missing subtrahend task of '10 to 6'. In the final example Bill used counting-down-from to solve a removed items task. Thus in some cases Bill might use a counting strategy. Nevertheless he has a range of robust non-count-by-ones strategies and was clearly at Stage 5: Facile Number Sequence.

## Scenario 4: Renae and Carol – Solution

Carol made two types of errors when attempting to count the collection of 13 counters. First, she makes a coordination error on each of her two attempts. On her first attempt she made 14 counts rather than 13 and on her second attempt she made 16 counts. Secondly, Carol made a number word sequence error on each of her two attempts. For these reasons Carol is judged to be at Stage 0: Emergent Counting.

## Scenario 5: Terry and Belinda – Solution

Belinda correctly counted a collection of 13 counters. On her first attempt to count a collection of 18 counters Belinda answered 17 and on her second attempt correctly answered 18. This indicates that Belinda was at least at Stage 1. Belinda was unable to solve tasks involving two screened collections, for example, she did not attempt to count from one or count-on to solve these tasks. This indicates that Belinda was not at Stage 2. Thus Belinda was at Stage 1: Perceptual Counting. On each of the two tasks involving two screened collections, when Belinda did not solve the task, Terry removed the screens and asked Belinda how many counters in all. Of interest is that in these cases Belinda also did not use counting. She did not seem to conceive of the two collections as being reorganized into one collection that could be counted. This is typical of some children at Stage 1, and also some children at Stage 0.

## Scenario 6: Terry and John – Solution

On two occasions John made a coordination error when attempting to count seven counters. He also made coordination errors when attempting to count 13 and 18 counters. John also made number word sequence errors in the teens when attempting to count collections of counters. This indicates that John was at Stage 0: Emergent Stage. When asked to count the counters which were arranged in a collection of five and a collection of two, John counted each collection separately. As in the case of Belinda in Scenario 5, John did not seem to be able to regard the counters as being reorganized into one collection which could be counted.

## Scenario 7: Tanya and Kelly – Solution

Kelly was able to solve additive tasks and Removed Items tasks but was not able to solve missing subtrahend tasks. Although Kelly did not count aloud or move her lips, the times taken to solve the tasks are consistent with her using a strategy involving counting-by-ones. Kelly may have been using her fingers under the desk to keep track of her counts. That she could not solve Missing Subtrahend tasks indicates that Kelly was less advanced than Stage 4. Whether Kelly counted-on or counted-from-one when solving the additive tasks is not apparent. Nevertheless her solution of four removed items tasks, and her explanation of her solution of '27 r 4' in terms of counting back provides sufficient evidence of advanced counting-by-ones strategies. Therefore Kelly is judged to be at Stage 3: Initial Number Sequence.

## Scenario 8: Joyce and Jed – Solution

On the basis of his performance on the various subtraction tasks Jed is no more than Stage 3. Although he did solve one missing subtrahend task he did not appear to have a general strategy for these tasks, for example, counting-down-to. In similar vein, Jed did not solve the two missing addend tasks and apparently did not use counting-up-to to attempt to solve these tasks. Jed also does not appear to have used counting-down-from to solve the removed items tasks. What is clear in the descriptions of Jed's solutions is that he used counting-on to solve the three additive tasks of 9 and 6, 8 and 5, and 9 and 3, and counted from one to solve 5 and 4. Why Jed counted-on on the last three tasks is not apparent. His use of counting-on was apparently spontaneous, in that Joyce did not appear to assist him. On the basis of his solutions of the last three additive tasks Jed is judged to be at Stage 3: Initial Number Sequence. That Jed solved three Removed Items tasks serves to strengthen the case for classifying him at Stage 3.

## Scenario 9: Kathryn and James – Solution

James was able to solve all three types of subtractive tasks as well as additive tasks, and correctly answered all of the tasks presented to him. James's solutions typically involved an advanced counting-by-ones strategy. James used counting-up-from to solve additive tasks, counting-up-to to solve missing addend tasks, counting-down-from to solve removed items tasks and counting-down-to to solve missing subtrahend tasks. Solving missing subtrahend tasks in this way is a strong indication that James was at Stage 4 rather than Stage 3. In solving the additive task of 5 and 4 the indication is that James did not use counting-by-ones because he answered quickly. Also, James may not have used counting-by-ones on the subtractive tasks of 3 r 1, 10 r 2 and 4 to 6, although it is difficult to discern counting-by-ones on tasks such as these where only one or two counts are required. In overall terms, there is little or no evidence to indicate that James is at Stage 5, that is, using a range of non-count-by-ones strategies. Therefore, on the basis of his facile use of advanced counting-by-ones strategies and, in particular, his use of counting-down-to to solve Missing Subtrahend tasks, James is judged to be at Stage 4: Intermediate Number Sequence.

## Scenario 10: Meg and Matthew – Solution

Matthew was clearly at least at Stage 1 because he successfully counted collections of 13, 15 and 18 counters. Matthew solved tasks involving two collections in cases where both collections were screened as well as in cases where only one collection was screened. In solving these tasks Matthew always counted from 'one' and thus he was not at Stage 3. Thus we can conclude that Matthew was at Stage 1 or 2. Matthew solved the following four additive tasks, 3 and 2, 5 and 4, 5 and 2, and 4 and 4 using the same strategy for each. This strategy involves raising fingers on his left hand to signify the counters in the first collection, then raising fingers on his right hand to signify the counters in the second collection and, finally, counting all of the raised fingers from one. Clearly, this strategy is viable in additive tasks where each addend is not greater than five. On each of the two tasks where one or both addends were greater than five, that is, 9 and 6 and 7 and 5, Matthew did not solve the task and did not appear to have any strategy available to him. Although Matthew may appear to have satisfied the criteria for Stage 2 because he solved four additive tasks in which one or both collections were screened, he is judged to be at Stage 1 rather than Stage 2. His strategy is referred to as 'building perceptual replacements', that is, his raised fingers are the perceptual items that have replaced the screened counters.

Matthew's strategy is a particular case of a strategy commonly used by young children to solve additive and subtractive tasks. In MR we have called this strategy 'counting forward from one three times'. Children typically use this strategy when they have materials which they can form into collections that signify addends, sums, minuends, and so on. Thus a child may use counters to solve 9 and 6 by counting out nine counters, then six counters and then counting all of the counters from one to 15. Similarly, a child may use counters to solve 12 – 9 by counting out 12 counters, then counting out and separating nine of the 12 counters, and then counting the remaining three. When counters are not available a child may use the fingers as in the case of Matthew but, typically, the strategy is viable for addition only when the two addends are no greater than 5, and viable for subtraction only when the minuend is no greater than 10. Children can become quite facile in their use of finger patterns as part of this strategy, for example children will raise fingers simultaneously to signify a number. In solving 5 and 3, for example, a child might raise five fingers on one hand simultaneously, then raise three fingers on the other hand simultaneously and then answer 'eight' without counting from one.

A comment on teaching can be made at this point. Consider a situation in which children are given the task of working out written additions or subtractions, for example, 5 and 2, 3 and 8, 14 – 7, 12 – 10, and so on, and are encouraged to use materials to work out their answers. In this situation children typically use the strategy of counting forward from one three times, that is, they count out a number of counters to signify the first number, and so on. Doing tasks of this kind is likely to encourage or reinforce the use of strategies involving perceptual counting and thus discourage advancement in terms of SEAL.

The strategy used by Matthew and the corresponding one for subtraction (as above) should not be confused with advanced counting-by-ones strategies in the particular case where the child uses fingers to keep track of the number of counts. Thus a child who solves an additive task such as 8 and 5 by raising five fingers in sequence to keep track of counting from 'nine' to 'thirteen' is using a Stage 3 strategy and this strategy should not be confused with the strategy of building perceptual replacements. In the case where the child raises five fingers prior to commencing their count this is also classified as a Stage 3 strategy. Similarly, children

at Stage 3 or Stage 4 will use finger patterns to keep track of counting when solving various types of subtractive tasks. As an example, a child at Stage 4 might solve the missing subtrahend task of 16 to 12 by raising four fingers in turn while counting from 'fifteen' to 'twelve'. It is conceivable that a child might solve the task of 8 and 5 involving two screened collections by counting from 'one' to 'eight' and then sequentially raising five fingers on one hand to keep track of counting from 'nine' to 'thirteen'. This strategy is not very common, probably because children who can use their fingers in this way usually count-on rather than count from one, that is, they are at Stage 3 at least. Nevertheless, the strategy just described would exemplify Stage 2: Figurative Counting.

## Scenario 11: Ivan and Rhett – Solution

In this scenario Rhett is presented with four additive tasks involving two collections, that is, 3 and 2, 5 and 2, 4 and 4, and 7 and 5. On the first task both collections were screened, whereas on the other three tasks the first mentioned collection only is screened. On the first task Rhett took a relatively long time to answer, that is, 30 seconds, and then answered 'four' rather than 'five'. It is reasonable to assume that because of Rhett's apparent difficulty with this task, Ivan decided that, on subsequent additive tasks, he would unscreen one of the two collections. Although Rhett did not provide any answer on the second task he correctly answered both the third and fourth tasks. It is clear that Rhett counted from one when solving the third and fourth tasks. This and his apparent lack of a strategy tc solve subtractive tasks indicates that he was no more than Stage 2.

Rhett's solutions to the third and fourth additive tasks are unusual to some extent. Both tasks involved a screened and an unscreened collection rather than two screened collections, and in both tasks Rhett first counted the unscreened collection rather than the screened collection. In continuing his count, to count the second screened collection, Rhett was able to keep track of four counts in the case of the third task, and seven counts in the case of the fourth task. For this reason Rhett is judged to be at Stage 2: Figurative Counting, rather than Stage 1.

## Scenario 12: Sandra and Ben – Solution

Ben solved a wide range of additive and subtractive tasks and, although his strategies were often not apparent, it was very clear that he was not counting from one. Thus Ben is at least Stage 3. In explaining the strategy he used to solve the missing subtrahend task of 12 to 9, Ben counted-down-to and used his fingers to keep track of three counts. This was indicative of a Stage 4 strategy. In the case of three of the additive tasks Ben indicated that he could use non-count-by-ones strategies. On the first task (5 and 4) he answered 'ten' and then quickly said 'nine'. He did not appear to use a counting strategy on this task and thus it is likely that he used the fact that five and five make ten to work out five and four. In explaining his solution to 9 and 6 he was aware that ten and six make sixteen, and in explaining his solution to 9 and 3 he was aware that ten and three make thirteen. In general terms Ben's solution strategies were not easily discernible. His advanced thinking was more apparent in his explanations than his solutions. Nevertheless he provided three indications of non-count-by-ones strategies and therefore is judged to be at Stage 5: Facile Number Sequence.

In the next chapter we continue to expand on SEAL and in particular on the Stage 5 child. We also present the second aspect of Part A of the LFIN by introducing the model for Tens and Ones.